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COMPARATIVE ANALYSIS OF SIMPLE MODELS OF DRYING OF THE LAYER OF FOREST COMBUSTIBLES, INCLUDING THE DATA OF EXPERIMENTS AND NATURAL OBSERVATIONS

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A comparative analysis of different simple models of drying of forest combustibles is considered. Results of the comparison of numerical calculations with experimental data and the data of natural observations are presented and discussed. A conclusion is drawn on the possibility of using simple mathematical models of drying the layer of forest combustibles in a new system of prediction of forest-fire hazard.

Introduction. As is known [1, 2], forest combustibles (FC) whose moisture content is less than certain critical values ignite and burn; therefore, mathematical modeling of drying the FC layer is an urgent problem for predicting the time of this process.

Clearly the prediction of a catastrophe is of importance for practicians only in the case where the time necessary for calculations is much smaller than the period of induction of the catastrophe [3]. Therefore, a new algorithm of modeling catastrophes, within which the probability of the catastrophe and times of its induction t^* and prediction t_p on a computer are estimated, is suggested in [3]. In the case where $t_p > t^*$, the mathematical model should be replaced by an expert system [3]. For effective prediction, the time t^* of forest-fire maturation of forest combustibles is bounded by the period from 5 a.m. (the time of sunrise in summer in Siberia) to 1–2 p.m. local time.

The present work is aimed at comparative analysis of the accuracy of different models of FC drying through comparison of results of the calculation with the known experimental data [1, 2, 4, 5] and estimation of the speed of computer simulation of the process.

The importance of this analysis is dictated by the necessity of choosing an optimum model which could be used in practice for predicting forest-fire hazard [3]. One of the requirements imposed upon the model is the rather quick obtaining of calculated data on moisture content. In the analytical review [6], the following information is given: as of 1995, in Russia there were 1807 forest husbandries and 7851 forest districts; each forest district had more than 100 planning quarters, which, in turn, consisted of 10–100 mapping units. Thus, one of the main requirements of the model is the small, compared to the time of catastrophe induction, expenditures of computer time for simultaneous prediction of forest-fire hazard on the territory throughout Russia.

The most comprehensive physicomathematical model of drying the FC layer is given in [7]. The model allows for a two-temperature nature of the medium, radiation from the sun, convective-conductive heat exchange with the near-ground layer of air, evaporation of free water and water bound with FC, partial pressure of water vapor in the FC layer and the near-ground layer of air, different types of FC in ground vegetation, and nonuniformity of the distribution of temperature and moisture content with space and time. The mathematical problem is reduced to a system of nonlinear partial hyperbolic-parabolic equations. Due to the complexity of this formulation for obtaining data on the moisture content of the layer of different FC, the calculation of one version requires several hours of computer time (Pentium-III-type PC), which, in light of what was stated above, is absolutely unacceptable and does not allow use of the given model in practice in the immediate future. In what follows, we analyze a number of simplified models of drying the FC layer in the above context.

One-Dimensional Mathematical Formulation of the Problem of Drying the FC Layer. The model is developed on the basis of [7, 8] with account for the following assumptions: (1) convective heat exchange of the FC

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layer with the near-ground layer of air is satisfactorily described by boundary conditions of third kind using the known equations of convective heat transfer; (2) the pressure P, temperature T, and density ρ of the gas phase in the FC layer coincide with the corresponding meteorological data (P_e, T_e, and ρ_e) for the given instant of time and given terrain; (3) radiation in the FC layer obeys the Bouguer–Lambert law; (4) evaporation of free water and water droplets which stuck to the elements of forest combustibles is described by the same Hertz–Knudsen law.

The equation of energy conservation and the equations for the volumetric fraction of dry organic substance and volumetric moisture content have the form [9]

$$\sum_{i=1}^{2} \rho_i \varphi_i C_{pi} \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_s \frac{\partial T_s}{\partial z}\right) + \frac{\partial q_{Rz}}{\partial z} - q_2 \frac{k'_{02}\rho_2\varphi_2}{\sqrt{T_s}}\left[\exp\left(-\frac{E}{RT_s}\right) - P_{2e}\right] - \alpha_v\,(T_s - T_e)\,, \tag{1}$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = 0\,, \tag{2}$$

$$\frac{d\varphi_2}{dt} = -\frac{k'_{02}\varphi_2}{\sqrt{T_s}}\left[\exp\left(-\frac{E}{RT_s}\right) - P_{2e}\right]. \tag{3}$$

The boundary conditions on the upper and lower boundaries of the FC layer are written as follows [9]:

$$-\lambda_s \left.\frac{\partial T_s}{\partial z}\right|_{z=h} = \alpha_e\,(T_{s,w} - T_e) + q_2 R_{2w} - \varphi_{s,w} q_{R,w}\,, \tag{4}$$

$$\lambda_s \left.\frac{\partial T_s}{\partial z}\right|_{z=0} = \alpha_0\,(T_{s0} - T_0) + q_{R,w}\,(1 - \varphi_{s,w})\exp\,(-k_1\rho_1 h)\,. \tag{5}$$

The initial conditions for temperature, volumetric fraction of the dry organic substance, and volumetric moisture content take on the form [9]

$$T_s\,(z)\big|_{t=0} = T_{s,in}\,(z)\,,\quad \varphi_1\,(z)\big|_{t=0} = \varphi_{1in}\,(z)\,,\quad \varphi_2\,(z)\big|_{t=0} = \varphi_{2in}\,(z)\,. \tag{6}$$

Zero-Dimensional Mathematical Formulation of the Problem of Drying the FC Layer. Using the method of equation averaging developed in the theory of thermal explosion [10], we obtained a system of ordinary differential equations where time is the only argument [11]:

$$\begin{aligned}(1 + a\varphi_2)\frac{d\theta_s}{d\tau} &= \frac{1}{\delta^2}\left\{-\mathrm{Bi}\,(\theta_s - \theta_e) - \frac{b\varphi_2}{\sqrt{1+\beta\theta_s}}\left[1 - \pi_e \exp\left(-\frac{\theta_s}{1+\beta\theta_s}\right)\right]\exp\frac{\theta_s}{1+\beta\theta_s} + \right.\\ &\left. + (\varphi_{1in} + \varphi_2)\,[c - d\,(1 + \beta\theta_s)^4] - \mathrm{Bi}_0\,(\theta_s - \theta_0) - (1 - \varphi_{1in} + \varphi_2)\,[c_0 - d_0\,(1 + \beta\theta_s)^4]\right\} + \\ &+ \frac{\bar{q}_{R,w}}{\bar{k}_1}\,(1 - \varphi_{s,w})\,(1 - \exp\,(-\bar{k}_1)) - \bar{\alpha}_v\,(\theta_s - \theta_e) - \frac{\varphi_2}{\sqrt{1+\beta\theta_s}}\left[1 - \pi_e \exp\left(-\frac{\theta_s}{1+\beta\theta_s}\right)\right]\exp\frac{\theta_s}{1+\beta\theta_s}\,,\end{aligned} \tag{7}$$

$$\frac{d\varphi_2}{\partial\tau} = -\frac{\gamma\varphi_2}{\sqrt{1+\beta\theta_s}}\left[1 - \pi_e \exp\left(-\frac{\theta_s}{1+\beta\theta_s}\right)\right]\exp\frac{\theta_s}{1+\beta\theta_s}\,, \tag{8}$$

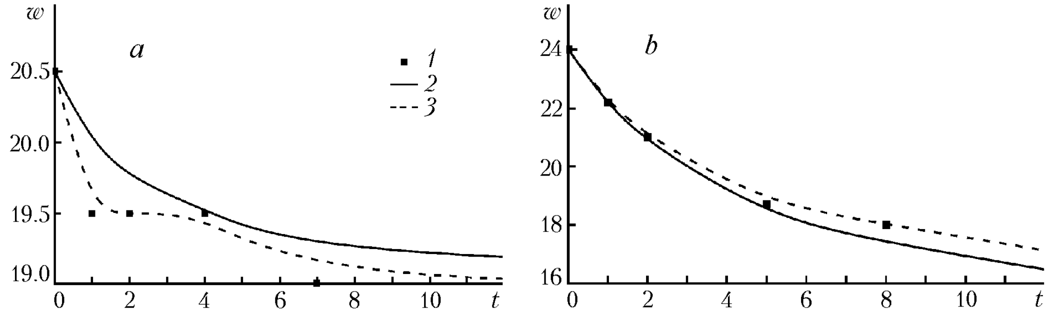

Fig. 1. Dependence of the moisture content of the FC layer on time according to numerical calculation and experimental data [4]: 1) experimental points; 2) results of numerical calculation; 3) spline- approximation by experimental points; a) T_e = 273 K, w_{in} = 20.5%, and π_e = 80%; b) 293, 24, and 60. w, %; t, h.

$$\varphi_2 \big|_{\tau=0} = \varphi_{2in}\,, \quad \theta_s \big|_{\tau=0} = 0\,. \tag{9}$$

Moreover, in [11], an approximate analytical formula is derived for determining the time of FC layer drying:

$$t_d = \tau_d t_{ch}\,, \quad \tau_d = \frac{\sqrt{(1-\pi_{e0})^2 + 2\left\{\theta'_{s0} - \pi_{e0}\right\}\dfrac{1}{\gamma}\ln\dfrac{\varphi_{2\acute{i}}}{\varphi_2^{*}}} - (1-\pi_{e0})}{\theta'_{s0} - \pi'_{e0}}\,. \tag{10}$$

Thus, we have two rather simple mathematical formulations and an approximate analytical formula for calculation of time of FC layer drying. Of interest is a comparative analysis of the accuracy of results obtained on the basis of these formulations and by formula (10) and also expenditure of computer time.

Data Base. For specific numerical calculations we must have a data base on the territory under control. For model calculations one can use the data base for the layer of forest combustibles of pine needles. The parameters of the pine needles are: ρ_1 = 500 kg/m^3; ρ_2 = 1000 kg/m^3; $T_{s,in}$ = 300 K; T_e = 300 K; λ_1 = 0.102 W/(m·K); λ_2 = 0.588 W/(m·K); q_2 = 2250 J/kg; R = 8.3144 J/(mole·K); k'_{02} = 6.03·10^5 K$^{1/2}$/sec, E/R = 4373 K; α_v = 239 J/(m^3·sec·K); h = 0.1 m; φ_{1in} = 0.06; C_{p1} = 1400 J/(kg·K); C_{p2} = 4180 J/(kg·K); P_e = 101,325 Pa; T_0 = 300 K; α = 0^o; α_e = 18.26 J/(m^2·sec·K); α_0 = 16.98 J/(m^2·sec·K); $q_{R,w}$ = 140 J/(m^2·sec); k_1 = 5.1 m^2/kg; π_e = 0.70. In calculations the data varied within certain limits.

Comparison of Results of the Numerical Calculation by the Model with the Data of Experiments and Natural Observations. Results of the experimental study of drying of forest combustibles are given in [4]. To create the medium with varying relative humidity of air, other factors being constant, exsiccators of sulfuric acid were used. The following versions of laboratory experiments were taken: (a) temperature of the surrounding medium was 273, 293, and 303 K; (b) relative humidity of air was 0, 20, 40, 60, 80, and 90–100%. The following time intervals (in hours) were used: 1, 2, 3, 4, 5, 7, 8, 24, and so on. Each experiment was performed on three samples, with a minimum error equal to zero and a maximum error of 1.15 [4].

Figure 1 presents results of the numerical calculations and experimental data [4] on moisture content. Figure 2 shows results of the numerical calculation of the dynamics of moisture content of the layer of forest combustibles of pine needles and the data of observations according to [1].

An analysis shows rather good agreement between results of the numerical calculation of the moisture content of the FC specimens and experimental data [4]. In most cases, the curves obtained numerically are rather close to the curves found from spline-approximation of experimental data. The best agreement is observed at a relative humidity of air close to 0% and a small initial value of moisture content of the forest combustibles. As the relative humidity and initial value of moisture content increase, the accuracy of the description of experimental data by numerical modeling decreases. Attention should be paid to Fig. 1a, where results of the comparison at T_e = 273 K and a relative humidity

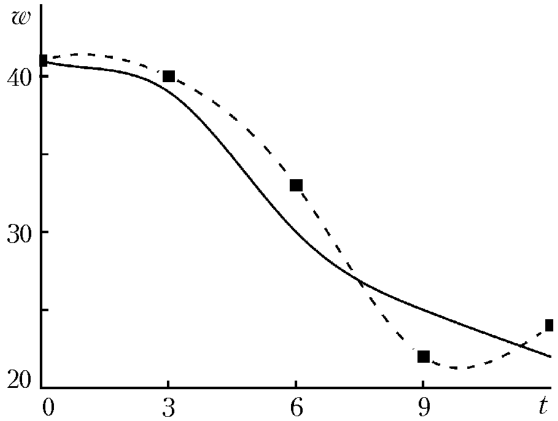

Fig. 2. Dynamics of the moisture content of the layer of forest combustibles of pine-needle litter after 5 a.m. according to numerical calculations and natural observations [1]. Notation of the curves as in Fig. 1. *w*, %; *t*, h.

of air of 80% are shown. In numerical modeling one fails to describe the situation where during 3 h the humidity of the sample in the exsiccator does not change. The time of calculation is limited to 12 h for two reasons. First, mathematical formulation (7)–(9) is applicable to the first half of the day when it is assumed that one makes forecasts, using the meteorological data, for the afternoon period (1–5 p.m., since, according to statistics, most forest fires emerge during this time). Second, mathematical model (7)–(9), as well as a more complex one-dimensional model, describes drying of the FC layer adequately only during the first 14 h of calculation; then the results are less adequate to reality.

The mean relative error does not exceed 15%, and, consequently, the curves of desorption obtained numerically do not fall beyond the limits of the confidence intervals of the results of the experiment [4]. Comparison with the data of natural observations [1] showed satisfactory agreement of the results (20–30%).

CONCLUSIONS

The results of the comparison of zero-dimensional and one-dimensional formulations indicate that the relative error does not exceed 10–15% [12]; this is in agreement with theoretical estimates of the accuracy of zero-dimensional models according to Khudyaev [13]. In [11], an approximate analytical formula for determining the time of drying of the FC layer is obtained and a comparative analysis of results found using the zero-dimensional formulation and the analytical formula is given. The analysis of the results presented in [11] showed that the mean relative error of analytical solution is 5.6% compared to numerical data. With account for the error of input data, all mathematical formulations given in this paper and the approximate analytical formula (10) describe the process of drying of the FC layer in the morning and noon satisfactorily.

A comparative analysis of the results by the time of execution of the program on a computer (iCeleron-533 MHz/128 Mb RAM) shows that in using the approximate analytical formula it amounts several seconds, for the zero-dimensional formulation it is half a minute, and for the one-dimensional formulation it is about 10 min. With account for the number of mapping units, we can draw the conclusion that only the approximate analytical formula can be used in prediction of forest-fire hazard within one forest husbandry. When the zero-dimensional and one-dimensional formulations are used and also in calculations for large territories covered by forest, one should employ multiprocessor computation systems and landscape parallelizing [14].

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NOTATION

z, coordinate reckoned from the ground surface normally to the underlying surface; *t*, time; T_S, temperature of the condensed (solid) phase (dry organic substance + free water and bound water); λ_S, thermal conductivity of the condensed phase in the FC layer; α_e and α_0, heat-transfer coefficients on the upper and lower boundaries of the layer; α, angle of slope to the horizon; T_{s0}, temperature of the FC layer on the lower boundary of the layer; T_e, surrounding temperature; T_0, temperature of the ground; $\alpha_v = \alpha_e S$, coefficient of volumetric convective heat transfer; S, specific surface area of the FC layer; ρ_i, C_{pi}, and φ_i, density, heat capacity, and volumetric fraction of the dry organic substance ($i = 1$) and free water and water bound by the dry organic substance ($i = 2$); $\lambda_s = \lambda_1\varphi_1 + \lambda_2\varphi_2$; k'_{02} and E, pre-exponential factor and energy of activation which characterize evaporation of free and bound water; w, moisture content; R, universal gas constant; $\varphi_{s,w} = \varphi_{1w} + \varphi_{2w}$, volumetric fraction of the condensed phase on the upper boundary of FC; φ_2^*, critical volumetric moisture content of the FC layer; $q_{R,w}$ and q_{Rz}, densities of the flux of resultant radiation on the phase interface and the flux of radiation penetrating to the FC layer; q_2, heat of evaporation of mass unit of water; k_1, coefficient of radiation decay in the FC layer; P_{2e}, partial pressure of water vapor; h, thickness of the FC layer; R_{2w}, mass flow rate of water evaporation on the layer–outer medium boundary; γ, dimensionless similarity criterion characterizing the rate of drying; δ^2, dimensionless criterion (analog of the Frank-Kamenetskii criterion); β, dimensionless quantity inverse to the energy of activation; Bi and Bi_0, Biot criteria characterizing intensity of heat exchange of the FC layer with the near-ground layer of air and ground; a, b, c, and d, dimensionless quantities characterizing volumetric heat capacity of water, thermal effect of water evaporation, influx of radiant energy, and emissivity, respectively; α_v, dimensionless volumetric coefficient of convective heat transfer; $\bar{q}_{R,w}$, dimensionless radiation heat flux; $\bar{k}_1$, dimensionless coefficient of radiation decay; π_e, relative humidity of air; $\theta_s = \frac{(T_s - T_{s,in})E}{RT_{s,in}^2}$, dimensionless temperature of the condensed phase of the FC layer; $\theta_0 = \frac{(T_0 - T_{s,in})E}{RT_{s,in}^2}$, dimensionless temperature of the ground; $\theta_e = \frac{(T_e - T_{s,in})E}{RT_{s,in}^2}$, dimensionless temperature of the surrounding medium; τ, dimensionless time; t_{ch}, characteristic time; $\pi'_{e0} = \left.\frac{d\pi_e(\tau)}{d\tau}\right|_{\tau=0}$; $\theta'_{s0} = \left.\frac{d\theta_s(\tau)}{d\tau}\right|_{\tau=0}$; $c_0 = c \exp(-k_1)$; $d_0 = d \exp(-k_1)$. Indices: e, surrounding medium; R, radiation; s, condensed (solid) phase; v, volume; w, wall $z = h$; 0, wall $z = 0$; in, initial; p, prediction; d, drying; ch, characteristic.

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